RA 04

1.12 Rules of inference with propositions

Rules of inference known to be valid arguments:

Rule of Inference	Name
p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \to q$ $q \to r$ $\therefore p \to r$	hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Example

Argument: $(r \vee w) \rightarrow c$
$\neg c$
$\therefore \neg w$

Proof:

1.	$(r \vee w) \rightarrow c$	hypothesis
2.	$\neg c$	hypothesis
3.	$\neg (r \vee w)$	Modus tollens 1,2
4.	$\neg r \wedge \neg w$	De Morgans Law, 3
5.	$\neg w \wedge \neg r$	Commutative Law, 4
6.	$\neg w$	Simplification, 5

1.13 Rules of inference with quantifiers

- A value that can be plugged in for variable is called an <u>element</u> of the domain.
  - An <u>arbitrary</u> element of the domain has no special properties other than those shared by all elements of the domain.
  - A <u>particular</u> element of the domain may have properties that are not shared by all the elements of the domain.
    - * if D is $\mathbb{R}$, then 3 is a particular element because it is odd.

- Any element defined in a hypothesis is <u>particular.</u>

- The rules of existential instantiation and universal instantiation replace a quantified variable with an element of the domain.
  - Universal Instantiation says that any element in the domain can substitute for a universally quantified variable.
  - Existential instantiation introduces a particular element to replace an existentially quantified variable.
    - * A new element with a new name must be used for each use of existential instantiation

- The rules of existential generalization and universal generalization replace an element of the domain with a quantified variable.
  - Existential generalization says that an element of the domain can be replaced by an existentially quantified variable
    - The rules ONLY APPLY TO NON-NESTED QUANTIFIERS

Rule of Inference	Name
c is an element (arbitrary or particular) $\forall x\, P(x)$ $\therefore P(c)$	Universal instantiation
c is an arbitrary element $P(c)$ $\therefore \forall x\, P(x)$	Universal generalization
$\exists x\, P(x)$ $\therefore$ (c is a particular element) $\wedge$ $P(c)$	Existential instantiation
c is an element (arbitrary or particular) $P(c)$ $\therefore \exists x\, P(x)$	Existential generalization

2.1 Mathematical definitions

- An integer x is **even** if there is an integer k such that $x = 2k$
- An iteger x is **odd** if there is an integer k such that $x = 2k+1$

- The **parity** of a number is whether the number is even or odd.
  - Two numbers have **same parity** if they are both even or both odd
  - Two numbers have **opposite parity** if one is even and one is odd

- A number r is **rational** if there exists integers x and y such that $y \neq 0$ and $r = \frac{x}{y}$.

- **Divides**
  - An integer x divides an integer y if and only if $x \neq 0$ and $y = kx$, for some integer x
  - The fact that x divides y is denoted $x \mid y$. If x does not divide y, the notation is $x \nmid y$
  - If x divides y, then y is a **multiple** of x, and x is a **factor** or **divisor** of y.

- An integer n is **prime** i.f.f. $n > 1$, and the only positive integers that divide n are 1 and n.
- An integer n is **composite** i.f.f. $n > 1$, and there is an integer m such that $1 < m < n$ and m divides n

- A real number x is **positive** i.f.f $x > 0$
- A real number x is **negative** i.f.f. $x < 0$
- A real number x is **nonnegative** i.f.f. $x \geq 0$
- A real number x is **nonpositive** i.f.f. $x \leq 0$